DIMENSIONAL ANALYSIS

- Dimensional Analysis is the practice of checking relations among physical quantities by identifying their dimensions.
- Isaac Newton (1686) called it the "*Great Principle of Similitude*"

- A dimensional equation uses natural units of nature such as mass, length, time, temperature or pressure. The units can be reduced or eliminated by scaling quantities.

Dimensional Analysis

Example: Express the distance 8.0 ft in terms of inches.

Conversion Factor for inches to feet:

1 ft = 12 in

$$8.0\ \text{ft} \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = \mathbf{96\ in}$$

Dimensional Analysis

Example: Convert the quantity 31,820 square miles (mi^2) to square meters (m^2) to 4 significant figures.

First, determine the conversion factors to be used.

Mile (mi) → Meter (m)

1 mi = 1.6093 km

1 km = 10^3 m

Dimensional Analysis

Example: Convert the quantity 31,820 square miles (mi^2) to square meters (m^2) to 4 significant figures.

1 mi = 1.6093 km

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$$31{,}820\ \text{mi}^2 \times \left(\frac{1.6093\ \text{km}}{1\ \text{mi}}\right)^2 \times \left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)^2 =$$

$$31{,}820\ \text{mi}^2 \times \left(\frac{2.5898\ \text{km}^2}{1\ \text{mi}^2}\right) \times \left(\frac{10^6\ \text{m}^2}{1\ \text{km}^2}\right) = 82407.4 \times 10^6\ \text{m}^2$$

$$= 8.241 \times 10^{10}\ \text{m}^2$$

Dimensional Analysis

Example: Convert the quantity 2.3 x 10^{-8} cm to nanometers (nm).

First, determine the conversion factors to be used.

Centimeter (cm) → Meter (m)

1 cm = 0.01 m = 1 x 10^{-2} m = 10^{-2} m

1 cm = 10^{-2} m

Meter (m) → Nanometer (nm)

1 nm = 1 x 10^{-9} m = 10^{-9} m

1 nm = 10^{-9} m

Dimensional Analysis

Example: Convert the quantity 2.3 x 10^{-8} cm to nanometers (nm).

1 cm = 10^{-2} m
1 nm = 10^{-9} m

Now we can set up the formula for the conversion:

Then cancel like terms that appear in the numerator and denominator:

$$2.3\times10^{-8}\,\text{cm}\times\left(\frac{10^{-2}\,\text{m}}{1\,\text{cm}}\right)\times\left(\frac{1\,\text{nm}}{10^{-9}\,\text{m}}\right)$$

Dimensional Analysis

Example: Convert the quantity 2.3 x 10^{-8} cm to nanometers (nm).

1 cm = 10^{-2} m
1 nm = 10^{-9} m

Leaving the correct unit; now determine numerical value:

$$2.3\times10^{-8}\,\text{cm}\times\left(\frac{10^{-2}\,\text{m}}{1\,\text{cm}}\right)\times\left(\frac{1\,\text{nm}}{10^{-9}\,\text{m}}\right)=0.23\ \text{nm}$$